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MODELING AND CALCULATION OF THE THERMAL DESTRUCTION OF A CYLINDRICAL SHELL

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An approximate analytical solution of the problem of nonlinear heat conduction in a cylindrical shell with allowance for the thermal destruction and removal of the material from the shell's surface as a result of the action of a high-temperature heat flux on it has been constructed. An example of numerical calculation has been given.

Physical Formulation of the Problem. We consider a nonstationary process of heat conduction in a thin-walled cylindrical shell at whose exterior surface a high-temperature gas flow arrives and heat exchange with the ambient medium is carried out on the interior surface by the Newton law. As a result of the high-temperature heating, the material is removed from the surface of the shell, which leads to its destruction. The density of the energy flux absorbed by the shell's surface is determined by the relations [1]

$$q = \lambda \frac{\partial u}{\partial n} + \rho H v ;$$

where n is the external normal to the shell's surface and v is the rate of destruction of the heated surface, which is equal to [1]

$$v = \tilde{v} \exp\left(-\frac{E}{u_{\text{w}}}\right);$$

here E is the quantity determined by the activation energy of the process of destruction; it is equal to the ratio of the activation energy of the process to the specific gas constant.

The problem in question belongs to the class of boundary-value problems of nonstationary heat conduction with a moving boundary [2–4].

The problem is solved in a nonlinear formulation where the temperature dependence of the thermophysical properties of the material of the shell is taken into account. Furthermore, the motion of the boundary of the destruction front of the shell is determined in the process of solution of the problem.

Mathematical Model of the Process. The physical formulation of the problem leads to the following mathematical model [5]:

$$\rho c (u) \frac{\partial u}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(r\lambda (u) \frac{\partial u}{\partial r}\right), \quad t > 0 , \quad R_0 < r < R (t) ; \tag{1}$$

$$u (r, 0) = T , \quad R_0 \le r \le R_0 + L ; \tag{2}$$

$$\lambda (u) \frac{\partial u}{\partial r}\bigg|_{r = R_0} = \alpha_0 (u_0 - U_{\text{m}}) , \quad t \ge 0 ; \tag{3}$$

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$$\lambda (u) \frac{\partial u}{\partial r}\bigg|_{r=R(t)} = \alpha_w (U_* - u_w) - \rho H v (u_w) , \quad t \geq 0 , \tag{4}$$

where $R(t)$ is the position of the destruction front of the shell, determined by the relation

$$R (t) = R_0 + L - \int_0^t v (u_w) \, dt .$$

Next, we introduce the functions $C(u, r) = \rho r c(u)$ and $\Lambda(u, r) = r\lambda(u)$ into consideration and write problem (1)–(4) in the form

$$C (u, r) \frac{\partial u}{\partial t} = \frac{\partial}{\partial r} \left(\Lambda (u, r) \frac{\partial u}{\partial r} \right), \quad t > 0 , \quad R_0 < r < R (t) ; \tag{5}$$

$$u (r, 0) = T , \quad R_0 \leq r \leq R_0 + L ; \tag{6}$$

$$\Lambda (u, r) \frac{\partial u}{\partial r}\bigg|_{r=R_0} = R_0 \alpha_0 (u_0 - U_m) , \quad t \geq 0 ; \tag{7}$$

$$\Lambda (u, r) \frac{\partial u}{\partial r}\bigg|_{r=R(t)} = R (t) [\alpha_w (U_* - u_w) - \rho H v (u_w)] , \quad t \geq 0 . \tag{8}$$

Construction of the Algorithm of Approximate Solution. The approximate analytical solution of initial boundary-value problem (5)–(8) will be sought with the use of the results of [6]. We carry out discretization of the time variable t by the points $t_k = kh$, $k = 1, 2, ...$ ($h > 0$ is a fairly small discretization step). The selection of the step determines the accuracy of approximation.

We replace the time variable in (5) by the finite-difference ratio

$$\frac{\partial u}{\partial t}\bigg|_{t=t_k} \approx \frac{u (r, t_k) - u (r, t_{k-1})}{h}$$

and carry out linearization of the problem on each time layer $t = t_k$, assuming that all the nonlinearities are known and have been found on the previous time layer $t = t_{k-1}$.

Having taken $u^{(k)}(r) = u(r, t_k)$, we introduce the notation

$$\Lambda^{(k)} (r) = \Lambda (u^{(k-1)} (r), r) , \quad C^{(k)} (r) = C (u^{(k-1)} (r), r) , \quad q^{(k)} = R_0 \alpha_0 (u_0^{(k-1)} - U_m) ,$$

$$R^{(k)} = R^{(k-1)} - v (u_w^{(k-1)}) \, h , \quad Q^{(k)} = R^{(k)} [\alpha_w (U_* - u_w^{(k-1)}) - \rho H v (u_w^{(k-1)})] .$$

Taking into account that $u^{(0)}(r) = T$, we write the difference-differential analog of problem (5)–(8):

$$-\frac{d}{dr} \left[\Lambda^{(k)} (r) \frac{du^{(k)}}{dr} \right] + \frac{1}{h} C^{(k)} (r) \, u^{(k)} (r) = \frac{1}{h} C^{(k)} (r) \, u^{(k-1)} (r) , \quad R_0 < r < R^{(k)} ; \tag{9}$$

$$\Lambda^{(k)} (r) \frac{du^{(k)}}{dr} = q^{(k)} \;\; \text{for} \;\; r = R_0 , \tag{10}$$

TABLE 1. Thermophysical Properties of the Thermal-Protection Material

u, K	300	500	700	900	1100	1300	1500	1700	1900
λ, W/(m·K)	1.50	1.56	1.70	1.86	2.06	2.30	2.58	2.90	3.28
c, J/(kg·K)	1500	1511	1534	1567	1611	1666	1732	1801	1890

$$\Lambda^{(k)}(r)\frac{du^{(k)}}{dr} = Q^{(k)} \text{ for } r = R^{(k)} . \tag{11}$$

At each iteration step, we will seek the solution $u^{(k)}(r)$ of boundary-value problem (9)–(11) in the form of expansion in a Fourier trigonometric series

$$u^{(k)}(r) = \sum_{n=0}^{\infty} \delta_n a_n^{(k)} X_n^{(k)}(r) , \quad \delta_n = \begin{cases} 0.5 , & n = 0 ; \\ 1 , & n > 0 , \end{cases} \tag{12}$$

in the system (complete and orthogonal on the interval $[R_0, R^{(k)}]$) of functions $X_n^{(k)}(r) = \cos \dfrac{n\pi(r - R_0)}{L^{(k)}}$, where $L^{(k)} = R^{(k)} - R_0$ is the shell thickness at $t = t_k$.

To solve problem (9)–(11) we use the method of finite integral transformations. We multiply both sides of Eq. (9) by $X_n^{(k)}(r)$ and integrate the resulting equality for r going from R_0 to $R^{(k)}$. Then, with account for boundary conditions (10) and (11), we obtain the infinite system of linear algebraic equations for the Fourier coefficients $a_n^{(k)}$ sought:

$$\sum_{m=0}^{\infty} G_{nm}^{(k)} \delta_m a_m^{(k)} = g_n^{(k)} , \quad n = 0, 1, \dots , \tag{13}$$

where

$$G_{nm}^{(k)} = \frac{\pi^2 nm}{4L^{(k)}} (\xi_{n-m}^{(k)} - \xi_{n+m}^{(k)}) + \frac{L^{(k)}}{4h} (\eta_{n-m}^{(k)} + \eta_{n+m}^{(k)}) ;$$

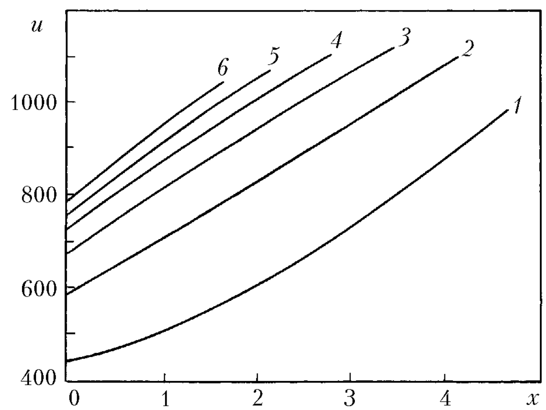
$$g_n^{(k)} = (-1)^n Q^{(k)} - q^{(k)} + \frac{L^{(k)}}{2h} \zeta_n^{(k)} ,$$

and $\xi_n^{(k)}$, $\eta_n^{(k)}$, and $\zeta_n^{(k)}$ are the Fourier coefficients of the functions $\Lambda^{(k)}(r)$, $C^{(k)}(r)$, and $\Theta^{(k)}(r) = C^{(k)}(r)u^{k-1}(r)$ in the orthogonal system $\left\{X_n^{(k)}(r)\right\}_{n=0}^{\infty}$.

The infinite system (13) can be solved by the reduction method [7] involving the replacement of an infinite system by the corresponding finite system of Nth order. The convergence of the reduction method for an infinite system of the form (13) has been substantiated in [8].

Results of Numerical Calculations. The above algorithm for calculation of the nonstationary temperature field in problem (1)–(4) has been employed for numerical calculations of the temperature distribution in the cylindrical layer of heat insulation of thickness L. The thermal-protection coating is applied to the cylindrical surface of a heat-insulated structure of prescribed temperature U_{m}. Assuming that thermal contact on this surface is nonideal, we can [9] model the condition of heat exchange on the interior surface of the thermal-protection cylindrical layer by boundary condition of the third kind (3).

We give an example of calculation of the temperature of the cylindrical thermal-protection layer subjected to short-duration ($0 \le t \le \tau$) intense heating for the following values of the parameters: τ = 60 sec, ρ = 1300 kg/m^3, α_0



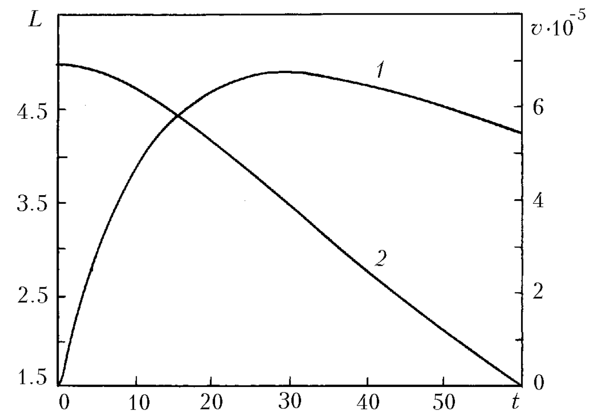


Fig. 1. Temperature distribution in the undestroyed part of the cylindrical thermal-protection layer at the instants of time: 1) t = 10; 2) 20; 3) 30; 4) 40; 5) 50; 6) 60 sec. u, K; $x = r - R_0$, mm.

Fig. 2. Evolution of the destruction rate (1) and the thickness of the undestroyed part of the thermal-protection layer (2). L, mm; v, m/sec; t, sec.

= 600 W/(m^2·K), α_w = 200 W/(m^2·K), U_m = 300 K, U_* = 3000 K, T = 300 K, E = 3000 K, H = 1.5 MJ/kg, $\tilde{v}$ = 0.001 m/sec, R_0 = 0.145 m, and L = 0.005 m.

The dependences of the thermal conductivity λ and the specific heat c on the temperature are given in Table 1 [10].

It is noteworthy that piecewise-linear approximation has been employed on each time layer t_k in the process of numerical calculations for construction of the functions $\Lambda^{(k)}(r)$ and $C^{(k)}(r)$ from the prescribed dependences $\lambda(u)$ and $c(u)$.

Figure 1 gives results of calculations of the temperature in the undestroyed part of the cylindrical thermal-protection layer at different instants of time. As the computations have shown, the temperature of the destroyed surface increases to the instant of time t = 26 sec, attaining a value of 1120 K and then decreases to 1030 K at t = 60 sec. Figure 2 plots the destruction rate and the thickness of the undestroyed part of the thermal-protection layer versus time. At the instant of time t = 60 sec, the thickness of the undestroyed part of the thermal-protection layer amounts to 30% of the initial thickness.

NOTATION

c, specific heat, J/(kg·K); H, thermal effect of the process of destruction, J/kg; h, time step, sec; L, thickness of the shell, m; R, radius of the shell, m; r, space coordinate along the radius of the cylinder, m; T, initial temperature of the shell, K; t, time, sec; U, temperature, K; u, temperature of the shell, K; v, rate, m/sec; α, heat-transfer coefficient, W/(m^2·K); λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; τ, fixed value of the time. Subscripts and superscripts: 0, interior shell surface; w, exterior shell surface; *, gas flow; m, ambient medium; k, iteration No.

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